

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

25th November 2023

Subject CM1A – Actuarial Mathematics (Paper A)

Time allowed: 3 Hours 15 Minutes (10.15 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1. Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. Mark allocations are shown in brackets.*
- 3. Attempt all questions beginning your answer to each question on a separate sheet.*
- 4. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1)

- i) Describe the benefits provided by typical health insurance contract. Describe the cashflows of contract from policyholder's perspective. (4)
- ii) A life insurance company provides the following types of insurance contracts:
- Pure Endowment
 - Endowment Assurance
 - Term Assurance

Describe the level of regular premium that will be levied for each type of contract in the ascending order for the lump sum benefit of INR 1,00,000 with policy term and premium paying term of 10 years offered to a particular policyholder aged 30 years exact. (3)
[7]

Q. 2) Describe the key steps involved in the process of developing actuarial models. [6]

Q. 3) A Life Insurance company has issued 40-years term assurance contracts with fixed sum assured of INR 1,50,00,000 to a group of 10,000 lives aged exactly 25 years. Premiums of INR 20,000 is payable annually in advance until the end of policy term or earlier death.

- i) What will be the gross premium reserve per policy after one year using the prospective method? (4)

Reserving Basis:

Interest Rate : 4% p.a. effective

Mortality : AM92 ultimate

Surrenders : None

Renewal Expense : INR 1,000 at the beginning of each policy year.

Renewal Commission : 1% of the annual premium.

- ii) The actual death experience of the insurance company pertaining to these policies during the first policy year was 5 deaths. What is the mortality profit or loss for the company? (4)

iii) The actual experience of the insurance company pertaining to these policies during the second policy year is as given below. Comment whether each of actual experience will lead to profit or loss for the Life Insurance company:

a) Actual renewal expenses of INR 1,200 per policy. (1)

b) Investment return earned on the underlying assets was 5.5% p.a. (1)

c) No surrender value is payable and 500 surrenders have occurred. (1)

d) Actual claim expenses of INR 1,00,000 per policy is payable. (1)

e) Actual number of deaths are 6 during second year. (1)

- f) Actual renewal commission of 1% of the annual premium. (1)
[14]
- Q. 4)** A 5 year unit linked endowment assurance contract is issued to a male aged 50 exact. The profit vector for 5 years are (-50, -10, 30, -35, 150).
- i) Calculate the reserves required at the end of years 1, 2 and 3 to zeroise the losses, assuming a rate of accumulation of 6% p.a. and mortality rate of 0.05 at each age. (4)
- ii) Without further calculations, compare the net present value of the profits before and after the zeroization. (2)
- iii) Describe the sources of income for life insurance Company from unit-linked contract. (5)
[11]
- Q. 5)** A loan of nominal amount INR 1,00,000 is to be issued bearing coupons payable quarterly in arrear at a rate of 5% per annum. Capital is to be redeemed at 103 on a single coupon date between 15 and 20 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.
- An investor who is liable to income tax at 20% and capital gains tax of 25% wishes to purchase the entire loan at the date of issue. Calculate the price which the investor should pay to ensure a net effective yield of at least 4% per annum. [8]
- Q. 6)** An analyst is valuing two companies, Hi-tech Corp and Fidelity Corp.
- Hi-tech Corp is assumed to pay its first annual dividend in exactly 6 years. It is assumed that the dividend will be Rs. 6 per share. After the sixth year, annual dividends are assumed to grow at 10% per annum compound for a further 6 years. Dividends are assumed to grow at 3% per annum compound in perpetuity thereafter.
- Fidelity Corp is expected to pay an annual dividend of Rs. 4 per share in exactly one year. Annual dividends are then expected to grow thereafter by 0.5% per annum compound in perpetuity.
- The analyst values dividends from both shares at the rate of interest 6% per annum effective in a discounted dividend model.
- i) Calculate the value, of Hi-tech Corp and Fidelity Corp both. (8)
- ii) There is a general rise in interest rates and the analyst decides it is appropriate to increase the valuation rate of interest to 7% p.a. effective. Calculate and show that the percentage fall in the value of Hi-tech Corp is greater than that in the value of Fidelity Corp. (6)
- iii) Explain your answer to part (ii) in terms of Duration. (2)
[16]
- Q. 7)** Describe the following probabilities and find their value using AM92 table
- i) $4|5q_{[45]+1}$ (2.5)

- ii) $2.5p_{[50]+0.5}$ assuming constant force of mortality between integer ages (2.5)
[5]

Q. 8) A man aged 45 years exact purchases two policies from an Insurer with the following features—

- I. First policy will pay a sum of INR 1,50,000 on his survival up to 15 years or on early death (Death benefits are payable at the end of year of death).
- II. Second policy will pay a whole life regular survival benefit of INR 10,000 per annum starting from the end of a deferment period of 15 years.

Write the expression of the random variable denoting the present value of the two benefits, their expected values and variances. [6]

Q. 9) A regular premium with profit 20-year endowment assurance policy is issued to a life aged 40 years exact with a sum assured of INR 5,00,000 payable at the end of the year of death or at maturity whichever is early. The policy is assumed to provide a simple bonus of 4% of the sum assured vesting at the end of each policy year. Along with Sum assured, one year bonus is payable if death occurs in the 1st policy year, similarly two years bonus if death occurs in 2nd policy year and so on. On survival till maturity, 20 years bonus would be payable along with sum assured.

Level premiums are payable quarterly in advance for the entire term, ceasing at earlier death.

Commission is payable at the time of premium payment at the rate of 25% of the premium for the first year and 5% thereafter.

Basis:

- Interest rate : 6% p.a.;
- Mortality : AM92 ultimate;
- Surrender : None;
- First year expenses : INR 2000 incurred on policy commencement and 15% of premium (incurred as and when premium is paid);
- Renewal expenses : 2% of each premium from the start of the 2nd year and INR 500 p.a., inflating at 1.923% p.a., from the start of the second policy year.

- i) Calculate the instalment premium payable under the policy. (15)
- ii) The policy has an option of receiving Maturity benefit in settlements instead of lump sum where it is paid in 5 annual instalments with guaranteed accruing annual interest rate at the rate of 5% p.a. 1st instalment is paid immediately on maturity.
- Calculate how much the Gross premium will change if the proportion of surviving policyholders at maturity opting for settlement is assumed to be 50% and no mortality and expense are charged during the settlement period. (5)
- iii) How will you compare the sensitivity of instalment premium with the change in policy term for a regular premium endowment assurance policy and a pure term assurance policy? (2)

[22]

Q. 10)

- i)** Name the criterion to measure the profitability of a capital project. (1)
 - ii)** Explain why the discounted payback period is a poorer decision criterion than net present value assuming that the investor is not short of capital. (2)
 - iii)** Why a lower coupon bond has higher Discount Mean Term (DMT)? (2)
- [5]**
