

# INSTITUTE OF ACTUARIES OF INDIA

## EXAMINATIONS

25<sup>th</sup> November 2023

**Subject CM2A – Financial Engineering and Loss Reserving (Paper A)**

**Time allowed: 3 Hours 15 Minutes (14.45 – 18.00 Hours)**

**Total Marks: 100**

### INSTRUCTIONS TO THE CANDIDATES

- 1. Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. Mark allocations are shown in brackets.*
- 3. Attempt all questions beginning your answer to each question on a separate sheet.*
- 4. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

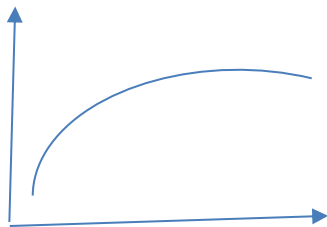
#### AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

**Q. 1)** What is Shiller's methodology about? Narrate criticisms made subsequently for the same. [3]

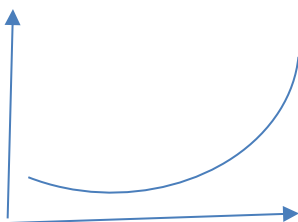
**Q. 2)** From below given graphs, identify the risk category of the investor and write the utility function condition with general reasoning:

i)



(2)

ii)



(2)

[4]

**Q. 3)** In context of current bond market, what could be the investment management strategy adopted by the Chief Investment Officer (CIO). [4]

**Q. 4)** Mr. Y has a quadratic utility function of the form  $U(w) = w - 15^{-7} w^2$ . It is Mr. Y's birthday today, he has invited Mr. Z to the party. Mr. Z is now looking for the birthday gift that would please Mr. Y. He is not sure how much amount he should budget for the gift. In the gift shop he likes three gifts. The cost of the gifts are Rs. 20,000, Rs. 40,000 and Rs. 50,000. Mr. Z likes all the gifts equally.

i) Calculate the expected cost of the gift and expected utility offered by those gifts to Mr. Y. (2)

ii) You know Mr. Y and his utility function, using this information which gift would you suggest to Mr. Z, such that Mr. Y is pleased. (4)

[6]

**Q. 5)** There are two investments X and Y. Investment Y provides a return  $3.5B\%$  where B is a Binomial random variable with parameters  $n=3, p=0.5$ . Investment X provides a return  $2P\%$  where P is a Poisson random variable with mean 3%. The benchmark return is 3% p.a.

i) Calculate the following :

a) Variance (2)

b) Downside semi-variance (4)

- ii) Without performing any further calculation, determine which investment an investor should choose, if their utility function is

$$u(x) = \begin{cases} -(1060 - x)^2, & x < 1060 \\ 0, & \text{elsewhere} \end{cases} \quad (2)$$

**[8]**

- Q. 6)** A market consists of three companies shares A, B and C with capitalisations of 2500, 4500 and 3000 crores respectively. Annual returns on the three shares have the following characteristics:

Return on company	Standard deviation
A	30%
B	20%
C	25%

The expected rate of return on the market portfolio is 12% p.a.

The correlation between the returns on each pair of distinct securities is 0.5.

The risk-free rate of return is 6% p.a. No adjustments to an investor's portfolio is possible within the year.

- i) Calculate the expected returns on Company A, B and C if the CAPM is assumed to hold. (8)

- ii) Calculate the covariance co-efficient of each of the securities with the Market. (2)

An investor wants to invest in Company A. She uses the Single index factor model (with the index equal to  $RM$ , the random return of the market portfolio) with the same expected returns and variances as in the CAPM.

- iii) Estimate the value of Alpha in the single factor model if the expected return of A is same that Expected return of A as per CAPM. (3)

- iv) Calculate the systematic and the specific risk that the investor would face if she invests in A. (4)

- v) Compare the correlation between A and B using the single index model & the CAPM and comment. (3)

**[20]**

- Q. 7)** Bank Regulator in a certain country has stipulated the following methodology to estimate the liability and solvency capital:

Liability estimation method : Calculated as the mean of liability.

Solvency estimation method : 100% of the standard deviation of the liability.

Interest rates are expected to follow the below mentioned rates with corresponding probabilities:

Interest rate	Probability
0.05	0.3
0.07	0.2
0.08	0.4
0.09	0.1

A Bank has the following Liability: Payment of 30,000 at the end of every year for the next 10 years. Other parameters are ignored for simplicity.

The Bank, however, estimates both using the following methodology:

**Liability estimation method:**

Calculate the mean return and estimate the value of liability using the mean return.

**Solvency estimation method:**

Step 1 : Calculate the standard deviation of the return.

Step 2 : Subtract standard deviation calculated in Step 1 from mean return.

Step 3 : Use the return from Step 2 to estimate a new liability.

Step 4 : Estimate solvency as the difference between liability from Step 3 and liability obtained using mean return.

- i) Estimate the value of the liability using the method specified by the Regulator. (3)
  - ii) Determine if the Bank is underestimating the liability. (3)
  - iii) Determine if the Bank is underestimating the solvency. (8)
  - iv) State one reason for the Bank to have followed the method. (1)
- [15]

- Q. 8)** A banker has issued 1500 one - year Fixed Deposits (FD) at start of the year. In case of deaths, the FD's are withdrawn prematurely. The annual premature claims have a compound negative binomial distribution, with parameters  $k = 0.4$  and  $p = 0.6$  and individual fixed deposit amounts have lognormal distribution with parameters  $\mu = 4.12$  and  $\sigma = 1.75$ .

The amount invested in the beginning of the years is Rs 150. It is expected that tenure of the pre-mature FD's would turnout to be 6 months. Calculate the minimum annual rate of interest the bank must earn throughout the year, if the maturity amount of the FD's is 175 and bank want total accumulation of Rs 50,000 with 99% probability.

[15]

- Q. 9)** With respect to options,

- i) Fill the table if the Greeks are positive or negative.

Greek	American call	European put
Delta		
Vega		
Theta		
Rho		
Lambda		

(5)

ii) Derive an expression for Theta of an option involving Gamma and Delta, under the Black-Scholes model. (4)

iii) Explain why a deep out of the money call option will have a rate of return close to risk free rate. (3)  
[12]

**Q. 10)** For a 2-period recombining binomial model for  $S_t$ , the price of a non-dividend paying stock at times  $t = 0, 1$ , and 2 is

$$S_t = \begin{cases} S_{t-1} \cdot u, & \text{with probability } p \\ S_{t-1} \cdot d, & \text{with probability } 1 - p \end{cases}$$

The state price deflator is :

$$A_t = \begin{cases} 0.761, & \text{when } S_t = S_{t-1} \cdot u \\ 1.522, & \text{when } S_t = S_{t-1} \cdot d \end{cases}$$

Risk free rate is 5% p.a. The price of a derivative that pays 1 at time 2, if  $S_2 < S_0$ .

i) Calculate the value of p. (5)

ii) Calculate the value of q. (5)

iii) Calculate the price at time 0 of a derivative that pays 1 at time 2 if  $S_2 > S_0$  using risk neutral probability measure. (3)  
[13]

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