INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

25th November 2023

Subject SP6 – Financial Derivatives

Time allowed: 3 Hours 15 Minutes (10.15 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.***
3. ***Attempt all questions beginning your answer to each question on a separate sheet.***
4. ***Mark allocations are shown in brackets.***
5. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1) Answer the following questions:

i) Without using the Black Scholes formula, show that European call and put options with the same strike and expiry have the same vega. (2)

ii) Suppose there are two American options, A and B where B has half the notional of A but is otherwise identical to A. Portfolio C consists of two contracts of B. Show that C is better than A. (2)

iii) What is the gamma of a vega neutral plain vanilla option? (2)

iv) In the Black Scholes model, as the risk-free interest rate increases, what happens to the European Call and Put values. (2)

v) What happens to the price of an American digital option when the volatility is increased, and the barrier has been breached. (2)

vi) Suppose an asset follows Brownian motion and there are no interest rates. What can be said about the relative prices of out of money American and European digital calls? (2)

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Q. 2) A stock X follows a geometric Brownian motion with time dependant volatility.

i) How will the time dependence affect the price of a down – and – out call? Distinguish the two cases where interest rates are zero and where interest rates are positive. (3)

ii) What happens, if the knock-out is determined by the forward rate for the same expiry instead of the spot price. (2)

[5]

Q. 3) K Bank has an exposure to \$100 million of debt issues by a company X. K Bank enters in to a CDS transaction with S Bank to hedge its debt exposure to company X. S Bank, in exchange for a premium, would fully compensate K Bank if X defaults. Assume that probabilities of default for K Bank, S Bank and X are 0.3%, 0.5% and 3.6% respectively. Further, defaults of the two banks and the company are correlated with coefficient of correlation 0.25.

i) If the default probability of security A is P(A) and default probability of security B is P(B) then show that their joint probability of default is given by

$$(Default\ Correlation\ of\ A\&B) * \sqrt{\left(P(A)\left(1-P(A)\right)\right)\left(P(B)\left(1-P(B)\right)\right)} + P(A)P(B)$$

(6)

ii) What is the probability that K Bank will suffer a credit loss in its exposure to company X? (2)

iii) Assuming K Bank suffers a credit loss, how much is the expected credit loss that it would be exposed to? (1)

iv) Compare the calculated expected loss with the expected loss if the default probabilities were independent of each other. (1)

[10]

Q. 4) Let S_t be a stock price process which follows a geometric Brownian motion with parameters μ and σ^2, and with stochastic differential equation:

$$dS_t = \left(\mu + \frac{\sigma^2}{2}\right) S_t dt + \sigma S_t dW_t$$

Where W_t is a Brownian motion. Let B_t be a risk-free asset whose price grows deterministically according to the formula $B_t = e^{rt}$ and also let $Z_t = B_t^{-1} S_t$ be the discounted stock price process.

Consider a dynamic portfolio (ϕ_t, ψ_t) consisting of ϕ_t units of S_t and ψ_t units of B_t, and let $X = f(S_T)$ be a path-independent claim on S_T.

Based on this information, answer the following:

i) Derive the stochastic differential equation for Z_t. (6)

ii) Explain what is meant by a self-financing and replicating strategy for X. Give the mathematical expressions wherever applicable. (6)

iii) Explain how the Cameron – Martin – Girsanov theorem and the Martingale Representation theorem can be used to construct a replication strategy for X. (6)

iv) Derive an expression for the stochastic differential equation for the value of the claim. (4)

[22]

Q. 5) Property investment company, CityScape Holdings, has a diversified portfolio of commercial properties. As of January 1, 2023, they own:

- A commercial building in the city center valued at ₹10 million.
- A shopping mall on the outskirts valued at ₹15 million.
- An office complex in the business district valued at ₹20 million.

CityScape is concerned about the potential decline in the property market over the next two years. They decide to use property derivatives based on different commercial property indices to hedge their exposure:

- The city centre building will be hedged using the City Commercial Property Index (CCPI), currently at 1,500 points.
- The shopping mall will be hedged using the Outskirts Retail Property Index (ORPI), currently at 1,200 points.
- The office complex will be hedged using the Business District Property Index (BDPI), currently at 2,000 points.

Each derivative contract will pay CityScape the difference between the index value at the start of the contract and the index value at the end of the contract, multiplied by the notional amount of the contract.

Each property derivative contract has an associated correction factor. This factor is used to adjust the payoff from the derivative to account for discrepancies between the index's movement and the actual property value change. The correction factor is applied to the calculated payoff from the derivative.

The correction factors are as follows:

- City Commercial Property Index (CCPI): 1.05
- Outskirts Retail Property Index (ORPI): 0.95
- Business District Property Index (BDPI): 1.10

i) Determine the notional amounts of the property derivative contracts for each property if CityScape wants to fully hedge its exposure. (4)

ii) At the end of the first year:

- CCPI has fallen to 1,450 points &
- ORPI has risen to 1,250 points &
- BDPI has fallen to 1,950 points.

Calculate the payoff from each property derivative contract. (4)

iii) CityScape had taken a loan of ₹5 million at an interest rate of 5% p.a. at the beginning of the year to finance some renovations in the office complex. If

- The value of the city centre building declined by 4% over the year &
- The shopping mall appreciated by 3% &
- The office complex declined by 5%.

Calculate the net gain or loss for CityScape after considering the change in the value of the properties, the payoff from the property derivatives, and the interest on the loan. (5)

[13]

Q. 6) A global investment firm maintains an extensive portfolio of European call options on a dividend-paying stock listed on the Bombay Stock Exchange. Given the prevailing economic conditions in India, the firm considers the following financial metrics for one particular option.

- The current stock price (S) stands at ₹ 10,000.
- The strike price (K) of the option is ₹ 10,500.
- The maturity period of the option is 6 months.
- The prevailing risk-free rate (r) in the Indian financial market is 5% per annum.
- The stock's volatility (σ) is registered at 22%.
- The dividend yield (q) for the stock is 2.5%.

Estimate the delta, gamma and vega of the option and hence predict the variation in option price when:

i) The underlying stock price undergoes an increment of 3%. (3)

ii) The volatility registers a fluctuation of 1.5%. (3)
[6]

Q. 7) A multinational financial institution is considering hedging its extensive interest rate exposure across multiple currencies using caps. The current 1-year LIBOR rate for USD is 3%, and the volatility of the forward rates is estimated to be 20%. The cap is set at a strike rate of 4% and has a maturity of 2 years. Additionally, the institution is also exposed to the EUR 1-year LIBOR rate, which is currently at 2%, with a volatility of 18%. The cap for EUR is set at a strike rate of 3.5% and also has a maturity of 2 years. Notional is ₹ 1 million.

i) Define the LIBOR market model and explain its significance in pricing interest rate derivatives. (2)

ii) Define the multi-currency LIBOR market model and explain its significance in pricing interest rate derivatives across different currencies. (2)

iii) Calculate the present value of the caplet for the first year for both USD and EUR using the Black's formula. (5)

iv) What will be the impact on the caplet's value for both currencies if the volatility of the forward rates increases. (1)

v) Explain how the multi-currency LIBOR market model can be extended to price cross-currency swaptions and the key considerations involved. (3)
[13]

Q. 8) A financial institution holds bonds from two companies: BRONCO and LFA LEX. The details of the bonds are as follows:

BRONCO bond:

- Par value: ₹100 million
- Market value: ₹95 million
- Maturity: 5 years
- Yield: 5%

LFA LEX bond:

- Par value: ₹50 million
- Market value: ₹48 million
- Maturity: 4 years
- Yield: 6%

The financial institution is considering entering 5-year Credit Default Swaps (CDS) to hedge its exposure to these bonds. The current CDS spreads for fixed annual payment contract are 200 basis points for Company BRONCO and 250 basis points for Company LFA LEX.

i) Calculate the annual premium payment for the CDS for both companies. (3)

ii) Assume a scenario where the credit event occurs for Company BRONCO 3 years into the contract and for Company LFA LEX 2 years into the contract. Calculate the payout from the seller of the CDS to the buyer, assuming a base recovery rate of 40% for both companies. (3)

After one year, if the CDS spread for Company BRONCO widens to 250 basis points and the CDS spread for Company LFA LEX narrows to 150 basis points, recalculate the annual premium payment for both CDS. Bond recovery rate is proportional to the CDS spread by following formula.

$$Recovery\ Rate\ (t) = \frac{CDS(T=0)}{CDS(T=t)} \times Base\ recovery\ rate$$

iii) Calculate the net impact on the financial institution's portfolio if both companies default after these changes in CDS spreads, considering the new premium payments and the payouts from the CDS. (5)

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Q. 9) You are an Investment Actuary, and you recently got an opportunity to attend a lecture by renowned derivative professor John Gull. In his lecture Prof Gull makes the following statement”

“With an appropriately chosen shape parameter ‘α’ a gamma distribution can be made sufficiently platykurtic and such a distribution can be quite useful under certain circumstances to model the price movement of the underlying for pricing options.”

i) What type of options in terms of their moneyness do you think can be better priced using platykurtic distributions as suggested by Prof Gull and why? (3)

ii) Discuss the advantages and disadvantages of the approach suggested by Prof Gull? (5)

[8]

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