Institute of Actuaries of India

Subject CM1A – Actuarial Mathematics (Paper A)

November 2023 Examination

INDICATIVE SOLUTION

Introduction
The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i) The benefits provided by typical health insurance contract are mentioned below:

- Hospital treatment either paid in full or part by the insurance company,
- Cash benefit by insurance company for hospital treatment,
- Such as fixed sum per day spent in hospital as an in-patient
- Payment towards any dentist fees or cost of specialist consultation or procedure
- Free Medical Check-up
- Payment towards ambulance service in case of emergency

The cash flows from the perspective of policyholder are mentioned below:

- Negative cash flow for payment of premium at the beginning of plan
- Or multiple negative cash flows in case premium is paid in instalments
- Positive cash flow in the event of claim in case of cash benefit
- No positive cash flow if the insurance company pays for hospital treatment directly [4]

ii) Pure Endowment provides benefit at the end of policy term on survival of policyholder.
Term Assurance provides benefit on death of policyholder within the policy term.
Endowment Assurance is a combination of above 2 benefits.

The probability of death of the policyholder aged 30 years exact within 10 years is lower than the survival till 10 years.

So, the premium in ascending order for the given insurance contracts will be as follows:

- Term Assurance
- Pure Endowment
- Endowment Assurance [3]

[7 Marks]

Solution 2:

The key steps involved in developing the actuarial models are:

- Develop a well-defined set of objectives that need to be met by the modelling process.
- Plan the modelling process and how the model will be validated.
- Collect and analyse the necessary data for the model.
- Define the parameters for the model and consider appropriate parameter values.
- Define the model initially by capturing the essence of the real-world system. Refining the level of detail in the model can come at a later stage.
- Involve experts on the real-world system you are trying to imitate to get feedback on the validity of the conceptual model.
- Decide on whether a simulation package or a general-purpose language is appropriate for the implementation of the model. Choose a statistically reliable random number generator that will perform adequately in the context of the complexity of the model.
- Write the computer program for the model.
- Debug the program to make sure it performs the intended operations in the model definition.
- Test the reasonableness of the output from the model.
- Review and carefully consider the appropriateness of the model in the light of small changes in input parameters.
- Analyse the output from the model.
- Ensure that any relevant professional guidance has been complied with.
- Communicate and document the results and the model.

[6 Marks]

Solution 3:

i) Gross Premium Reserve per policy after 1 year will be:

$${}_1V = 15000000\, A^{1}_{26:\overline{39}|} + 1000\ddot{a}_{26:\overline{39}|} - 0.99 * \ 20000\ \ddot{a}_{26:\overline{39}|}$$

$$A^{1}_{26:\overline{39}|} = \ A_{26:\overline{39}|} - \ v^{39}\, {}_{39}p_{26} \ = \ 0.03733$$

$$\ddot{a}_{26:\overline{39}|} \ = 20.035$$

$${}_1V = 15000000\ (0.03733) - 18800\ (20.035) = INR\ 183292$$ [4]

ii) Death Strain at Risk (DSAR) = 15000000 – 183292 = 14816708

Expected Death Strain = $q_{25} * DSAR * 10000 = 0.000566 * \ 14816708 * 10000$

Expected Death Strain = 83862567.28

Actual Death Strain = 5 * DSAR = 74083540

Mortality Profit = Expected Death Strain – Actual Death Strain = INR 9779027.28 [4]

iii)

a) The actual renewal expenses were higher than renewal expenses allowed for in the reserving basis, therefore there will be loss on renewal expenses. [1]

b) The actual interest earned is higher than allowance made in the reserving. So, the company will earn more and will lead to profit on account of investments. [1]

c) The surrenders will lead to release of reserves as no allowance for surrenders is made in the reserving basis and no surrender is payable. This will lead to profits for the company. [1]

d) No allowance for claim expenses is made in the reserving and the actual claim expenses incurred will lead to loss for the company. [1]

e) The company expects approximately 5.6 death (0.000567 * 9995) which is lower than actual deaths of 6 deaths. This will lead to mortality loss for the company as expected death strain is lower than actual death strain. [1]

f) Renewal commission is line with the reserving assumption, and this will neither lead to profit nor loss for the company. [1]

[14 Marks]

Solution 4:

i) The reserves required at the end of year 1, 2 and 3 are:

$${}_3V = \frac{35}{1.06} = 33.019$$

$${}_2V = \frac{1}{1.06}\{-30\ + (1 - 0.05) * 33.019\} = 1.291$$

$${}_1V = \frac{1}{1.06}\{\ 10\ + (1 - 0.05) * 1.291\} = 10.591$$ [4]

ii) In order to zeroise the negative non-unit cash flows, the reserves are created which leads to deferment of profits to later period.

Additionally, the discount rate used to calculate NPV is usually higher than interest rate for accumulation of non-unit fund.

As a result, NPV of profits before zeroization is higher than NPV of profits after zeroization. [2]

iii)

- Bid Offer Spread: The bid price is price at which the units are repurchased from policyholder and offer price is the price at which the units are purchased by the policyholder. The difference between the above 2 prices is known as bid offer spread. The offer price is higher than bid price to allow the life insurance company to generate a revenue stream.

- Unallocated Premium: The percentage of premium which is not allocated to unit fund becomes the income for insurance company.
- Fund Management Charge: The fund management charge expressed as percentage of unit fund is used to cover the investment related expenses incurred by the company in managing the unit fund.
- Policy Fee: The fees generally fixed in monetary terms is deducted from unit fund to cover the administrative expenses of the insurance company.
- Other Charges: These are charges to cover the cost of providing any additional non-unit benefits like guaranteed death benefit or maturity benefit. **[5]**

[11 Marks]

Solution 5:

Check if there would be a capital gain

$[1+i^{(4)}/4]^4=1.04 \Rightarrow i^{(4)}=0.039414$

Now

$g^*(1-t_1)=0.05/1.03^*0.80=0.038835$

- ⇨ $i^{(4)} > g^*(1-t_1)$
- ⇨ There is a capital gain on contract
- ⇨ Assuming that the instrument is redeemed as late as possible i.e. after 20 years, to obtain minimum yield

The price of stock would be

$P=100000^*0.05^*0.80^*\ a_{\overline{20}|}^{(4)}$

$+(103000-0.25(103000-P))v^{20}$ at 4%

- ⇨ $P=(4000^*a_{\overline{20}|}^{(4)}\ +77250v^{20})/(1-0.25v^{29})$
- ⇨ P=102072.25

[8 Marks]

Solution 6:

i) Valuation of Hi-tech shares

$V_H = 6v^6 + 6X1.1v^7 + 6X1.1^2v^8 + \ldots\ldots + 6X1.1^6v^{12} + 6X1.1^6X1.03v^{13} + 6X1.1^6X1.03^2v^{14} + \ldots\ldots$ (1.5)

$V_H=6v^5X[(1-1.1^7v^7)/(1-1.1v)] + 6X1.1^6x1.03v^{13}X(1/(1-1.03v))$

Putting the values of v, v^6, v^7, v^{13}, 1.1^6 and 1.1^7

$V_H=214.543$

Valuation of Fidelity shares

$V_F=4v+4X1.005v^2+4x1.005^2V^3+\ldots\ldots$

V_F=4vX(1/(1-1.005v))

=72.727 [8]

ii) Revised value of Fidelity is the same with v =1/1.07

Therefore V_F=61.538

So percentage change in the value is = -15.38%

Revised value of High-tech is

V_H=6X0.666342X[(1-1.948717X0.622750)/(1-1.1x0.934579)]+6X1.1^{6}X1.03X0.414964X(1/(1-1.03X934579))

=30.45408+121.52720=151.981

Therefore percentage change in value is = -29.16% [6]

iii) The higher change in value of Hi-Tech is due to the fact that its cash flows are much later therefore their interest rate volatility and duration are higher. [2]

[16 Marks]

Solution 7:

i) Probability that a life of age 46 years now and who has entered into select population at the age of 45 years, will live for next 4 years and die thereafter within next 5 years.

$_{4|5}q_{[45]+1}$
$= {}_4p_{[45]+1} \text{ X } {}_5q_{50}$
$= (l_{50} / l_{[45]+1}) * (1 - l_{55} / l_{50})$
= (9712.0728 / 9786.3162) * (1 – 9557.8179 / 9712.0728)
= 0.992414 X .015883
= 0.015762 Ans. [2.5]

ii) Probability that a life of age 50.5 years now and who has entered into select population at the age of 50 years, will survive for next 2.5 years

$_{2.5}p_{[50]+0.5}$
$= {}_{0.5}p_{[50]+0.5} \text{ X } {}_2p_{[50]+1}$
$= (p_{[50]})^{0.5} \text{ X } {}_2p_{[50]+1}$ (using constant force of mortality assumption)
$= (1 - q_{[50]})^{0.5} \text{ X } (1 - q_{[50]+1}) \text{ X } (1 - q_{52})$
$= (1 - .001971)^{0.5}$ X (1 - .002732) X (1 - .003152)
= .999014 X .997268 X .996848
= .993144 Ans. [2.5]

[5 Marks]

Solution 8:

Random variable denoting PV of benefits under policy a :

This is an endowment assurance policy with sum assured of Rs. 150000 and policy term of 15 years.

If K_{45} is the random variable denoting the curtate lifetime for a person aged 45;
Then Random Variable (PV of benefits) =
H = 150000 v^{K45+1}, if K_{45} < 15
= 150000 v^{15}, if K_{45} >= 15

Or

$150000\ v^{\min(K_{45}+1,15)}$

EPV(H) = 150000 $A_{45:\overline{15|}}$

Variance(H) = 150000^2 X [$({}^2A_{45:\overline{15|}}$ - $(A_{45:\overline{15|}})^2]$

If $A_{45:\overline{15|}}$ is calculated at an interest rate = i then ${}^2A_{45:\overline{15|}}$ is calculated at an interest rate = $(1 + i)^2$ - 1

Random variable denoting PV of benefits under policy b :

This is a deferred annuity policy paying regular annuity of Rs. 10000 p.a. in advance after the deferment term of 15 years till the policyholder survives.

The Random Variable (PV of benefits) =

Y = 0, if $K_{45} < 15$

= $10000\ v^{15}\ \text{adue}_{\overline{K_{45}+1-15|}}$, if $K_{45} >= 15$

Or

$10000\ v^{15}\ \text{adue}_{\overline{\max\{K_x+1-15,0\}|}}$

EPV(Y) = 10000 ${}_{15|}\text{adue}_{45}$

Variance(Y) = $10000^2\ (1 / d^2)$ X [$v^{30}\ {}_{15}q_{45} + {}_{15|}{}^2A_{45} - (v^{15}\ {}_{15}q_{45} + {}_{15|}A_{45})^2$]

Where if ${}_{15|}A_{45}$ is calculated at an interest rate = i then ${}_{15|}{}^2A_{45}$ is calculated at an interest rate = $(1 + i)^2 - 1$

[6 Marks]

Solution 9:

i) EPV Premium = $4P\text{adue}^{(4)}_{40:\overline{20|}}$

= $4P(\text{adue}_{40:\overline{20|}} - 3/8\{1 - v^{20}$ X $(l_{60} / l_{40})\}$

= $4P(11.998 - 3/8\ \{1 - 1.06^{-20}(9287.2164/9856.2863)\}$

= 4P X 11.73318

Bonus payable per annum = 4% of 500000 = 20000

EPV Benefit = (500000 – 20000) X $A^1_{40:\overline{20|}}$ + 20000 X $(IA)^1_{40:\overline{20|}}$

+ (500000 + 20 X 20000) X $A_{40:\overline{20|}}^{\ \ 1}$

= 480000 X $\{A_{40} - v^{20}(l_{60}/l_{40})A_{60}\}$

+ 20000 X $[(IA)_{40} - v^{20}(l_{60}/l_{40})\ \{(IA)_{60} + 20\ A_{60})\}]$

+ 900000 X $v^{20}(l_{60}/l_{40})$

= 480000 X $\{0.12313 - 1.06^{-20}(9287.2164/9856.2863)$ X $0.32692\}$

+ 20000 X $\{3.85435 - 1.06^{-20}$X$(9287.2164/9856.2863)$X$(5.46572 + 20$ X $0.32692)\}$

+ 900000 X 1.06^{-20} (9287.2164/9856.2863)

= 12998.50 + 6550.28 + 264421.92

= 283971

EPV Commission = 0.25 X $4P\text{adue}^{(4)}_{40:\overline{1|}}$ + 0.05 X 4P X $v(l_{41} / l_{40})$ X $\text{adue}^{(4)}_{41:\overline{19|}}$

= 0.25 X 4P X $(\text{adue}_{40} - 3/8 - v(l_{41}/l_{40})(\text{adue}_{41} - 3/8))$

+ 0.05 X 4P X $\{1.06^{(-1)}$ X (9847.051 / 9856.2863)

X $\{\text{adue}_{41:\overline{19|}} - (3/8)$ X $(1 - v^{19}$ X $l_{60} / l_{41})\}$

= 0.25 X 4P $\{15.491 - 3/8 - 1.06^{(-1)}(9847.051/9856.2863)(15.375 - 3/8)\}$

+ 0.05 X 4P X $1.06^{(-1)}$ X (9847.051 / 9856.2863)

X {11.669 – (3/8) X (1 – $1.06^{(-19)}$ X (9287.2164 / 9847.051)}

= 0.25 X 4P X 0.978316
+ 0.05 X 4P X 0.942512 X (11.669 - 0.258104)
= 0.25 X 4P X 0.978316 + 0.05 X 4P x 10.7549
= 4P X 0.782324

Effective interest rate for Renewal Fixed expense = (i – f)/(1+f)
=(0.06 – 0.01923) / (1 + 0.01923) = 0.04

EPV Expenses:
= 2000 + 500 $a_{40:19\rceil}$(@ 4% p.a.)
+ 4P$\ddot{a}^{(4)}_{40:1\rceil}$ X 0.15 + 4P$a^{(4)}_{40:19\rceil}$ X 0.02

= 2000 + 500($\ddot{a}_{40:20\rceil}$ - 1) (@4% p.a.) + 0.15 X 4P X 0.978316
+ 0.02 X 4P X 10.7549 (Using values from the calculation of commission)

= 2000 + 500(13.927 – 1) + 4P X 0.3618454
= 2000 + 6463.50 + 4P X 0.3618454
= 8463.50 + 4P X 0.3618454

From Principle of equivalence:
EPV Premium = EPV Benefits + EPV Expenses + EPV Commission
4P X 11.73318 = 283970.70 + 8463.50 + 4P X 0.3618454 + 4P X 0.782324
4P X (11.73318 – 0.3618454– 0.782324) = 292434.20
4P = 292434.20 / 10.589
4P = 27616.79

Instalment premium (P) = 6904 [15]

ii) Maturity benefit = SA + accrued bonus for 20 years
= 500000 + 20 X 20000
= 900000
Effective interest rate for discounting the instalment = 1.06 / 1.05 – 1 = 0.009524

PV of instalments at maturity

= (900000 / 5) X (1.009524) X (1 – $1.009524^{(-5)}$) / 0.009524
= 180000 X 4.906544 = 883178

Value of maturity benefits
= 0.5 X 264422 + 0.5 X 883178 X 264422 / 900000
= 261951

So instalment premium will decrease by = (264422 – 261951) / (4$\ddot{a}^{(4)}_{40:20\rceil}$)
= 2471 / (4 X 11.73318) = 52.65 [5]

iii) The difference is due to the difference in the interest rate at which instalment grows and the rate at which it is discounted. If interest were accrued @6%, there would be no difference in Premium.

Typically the premium chargeable under an endowment assurance policy decreases with the increase in policy term due to delay in maturity payment, while in term assurance, the premium increases with the increase in term due to higher mortality cost involved for longer duration and for older ages. [2]

[22 Marks]

Solution 10:

i) The criterion to measure profitability of a capital project are

1. NPV- Net Present Value
2. IRR- Internal rate of Return
3. DPP- Discounted Payback Period [1]

ii) If the investor is not short of capital the DPP is possibly not a good criterion to measure profitability of a project as it only indicates when a project would come into profitability and does not indicate how profitable the project is. [2]

iii) A lower coupon bond would have higher weightage towards redemption as compared with a higher coupon bond. Therefore a lower coupon bond will have higher DMT [2]

[5 Marks]

****