

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

29th November 2023

**Subject CM2B – Financial Engineering and Loss
Reserving (Paper B)**

Time allowed: 1 Hour 45 Minutes (14.45 – 16.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1. Mark allocations are shown in brackets.*
- 2. Attempt all questions beginning your answer to each question in the template provided.*
- 3. Attempt all sub-parts of the question in the template provided only, unless otherwise instructed to do so.*
- 4. The working of each part of the question should be on a separate tab (sheet). For example, question 1(i) should be worked out within the tab (sheet) with name 1(i) of worksheet.*
- 5. Where possible, summarize your data used & assumptions made (if any) in a separate tab.*
- 6. Do save your work in solution template on a regular basis.*
- 7. All the detailed guidelines are available on exam screen.*
- 8. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you. You are requested to save and submit the work before leaving the examination premises.

Q. 1) Forward rates are given as data in template for each of the next 20 years.

- i) Calculate the Price and the Gross Redemption Yield (GRY) of a 12-year Zero Coupon Government bond of face value 100, eight years from now, if there is no change in the term structure. (7)

Two investors purchase 20-year Corporate Bond issued by XYZ corp. Face value of the bond is Rs 100. Corporate spread of XYZ corp. is 100 bps over Govt. GRY (viz. 0.01 is to be added to the Govt. GRY). Corporate bond spread of XYZ corp is not constant but varies continuously by following a Cox Ingersoll Ross (CIR) stochastic process over the 20 years.

Observed Corporate spread at any point in time is used to estimate the price of the bond at the time by adding the spread to the then Government bond GRY of the equivalent term.

- ii) Calculate a sample path of Corporate bond spread at each of the next 20 years using the CIR model ($\alpha = 0.15$, $\mu = 0.01$, volatility = 0.05) and the uniform random variable given in the excel template. (8)

First investor keeps the corporate bond till maturity and receives 90% of face value of the bond as recovery at the end of 20 years. Second investor sells the bond at the end of 8 years and reinvests the amount in the risk free Zero Coupon Govt. bond for the remaining 12 years.

- iii) Find the Internal Rate of Return (IRR) obtained by both the investors using the results of part (i) if required. (17)

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Q. 2) Following shows cumulative incurred claims data, by year of accident and reporting development for a portfolio of domestic household insurance policies:

Cumulative incurred claims (Rs 000)		Development Year			
		0	1	2	3
Accident year	2019	729	817	878	920
	2020	826	887	953	
	2021	897	998		
	2022	921			

Inflation rates over the 12 month upto the middle of the given year are:

Year	Inflation Rates
2019	0.070
2020	0.065
2021	0.080
2022	0.050

The corresponding cumulative number of reported claims by year of accident and reporting development are as follows:

Cumulative no. of reported claims (Rs 000)		Development Year			
		0	1	2	3
Accident year	2019	53	58	60	64
	2020	55	59	62	
	2021	61	66		
	2022	60			

- i) The claim paid to date are Rs. 26,40,000. Use average cost per claim method with simple average grossing up factors in the template model to calculate an estimate of the outstanding claim amount of these policies for claims arising during these accident years. (18)

- ii) State assumptions used. (2)

[20]

- Q. 3)** An insurance has a portfolio of policies, for which claims occur as Poisson process of 30 claims per year.

The claim amount follows a generalised three parameter Pareto distribution with parameter $k = 4$, $\lambda = 250$ and $\alpha = 3$. The insurer includes a premium loading of 20% in its premium for this portfolio. Insurer has an initial capital of Rs. 5000.

It plans to use reinsurance to use the capital efficiently. It has received few offers. Help the insurance company to evaluate the below offers, such that the probability of ruin is limited to 2%.

Given that aggregate claims are approximately normal distributed.

Name of re-insurer	% of loss to be re-insured	Loading used by re-insurer
x	10	25
y	20	30
z	30	45
a	40	20
b	35	50
c	25	40
d	30	40
e	20	25

[15]

- Q. 4)** A non-dividend paying stock has price Rs. 125. Risk free rate of interest is 5% p.a. Assume volatility of 15% p.a. Strike price is Rs. 126.

- i) For time to maturity of 6 months, calculate the price of an European call. (5)
- ii) For similar data as part (i), calculate the price of an American call and explain your answer. (2)
- iii) For time to maturity of 6 months, calculate the price of a European put. (2)
- iv) Assume that the European call and European put are available with various times to maturity - end of the day of each month. Plot the prices of these options in a graph with times to maturity of 1 month, 2 month, 3 month ... 12 month. (8)

[17]

- Q. 5)** An asset gives a random rate of return “R” over a year. It is believed “R” has a distribution that is a combination of two normal distributions X and Y. X is supposed to have a normal distribution with mean 0% and standard deviation 13% and Y is a normal distribution with mean 15% and standard deviation 13%.

$$R = 0.75X + 0.25Y$$

- i) You are given 100 simulations from a uniform random variable. Using this, simulate 100 random variables from the two normal distributions that make up random variable R. (5)
- ii) Calculate the mean and Standard Deviation (SD) of R based on $R = 0.75X + 0.25Y$. (5)
- iii) By simulating 100 values from R using the same set of uniform random variables, calculate the shortfall probability for R against a benchmark of 0% and -15% using simulated values and verify using formula method. (6)

[16]
